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MATHEMATICAL ANALYSIS OF INFLOW PERFORMANCE FOR MULTIPHASE FLOW RESERVOIRS

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Abstract

The importance of Inflow Performance Relationship (IPR) models to the oil and gas industry cannot be over-emphasized. IPR models are very essential in predicting future production from hydrocarbon reservoirs. Accurate prediction of future production from hydrocarbon reservoirs is very essential as it influences field's viability and economic analysis. For years, attempts had been made by many authors to develop models suitable for this purpose. However, they have not been so successful as most the developed models have been found wanting in accuracy. Hence, there is a need for improved and accurate models. Therefore, the objective of this research is to introduce IPR models that can be used to predict production from hydrocarbon reservoirs with high degree of confidence. The pseudo-steady state solution of the Partial Differential Equation (PDE) governing multiphase flow in homogenous and isotropic porous media was obtained via Laplace Transform. Furthermore, the obtained solution was expanded using Taylor's series expansion method in order to obtain a form that is suitable for forecasting production from hydrocarbon reservoirs. By considering different number of terms in the Taylor's series form of the solution, five different Inflow Performance Relationship (IPR) models were obtained. Furthermore, performance analysis was carried out using statistical metrics to ascertain the reliability of the developed models. The result of this analysis shows that the developed models perform better than Vogel and Wiggins models, the two widely celebrated models in the oil and gas industry.

Keywords: Mathematical Analysis; Inflow Performance; Multiphase Flow; Production Optimization.

1.0 Introduction

The inflow performance relationship (IPR) has long been shown to be essential in monitoring and optimizing the producing life of a reservoir. Consequently, attempts had been made by several authors to develop such relationship for different reservoirs with different fluids compositions and flow regimes.

When calculating the productivity of oil wells, it is commonly assumed that flow into a well is directly proportional to the pressure differential between the reservoir and the wellbore. However, **[6]** pointed out that this relationship is not expected to hold when two-phase flow exists in a reservoir. This, they proved by presenting theoretical calculations to show that curves rather than straight lines result from two-phase flow. **[7]** proposed methods of well analysis that could utilize the whole curve of producing rates plotted against intake pressures. Thus, he termed it a complete graph of inflow performance relationship (IPR) of a well. **[11]** developed one of the earliest IPRs using a computer program based on Weller's assumptions for solution gas drive reservoirs to predict inflow performance curves. Weller's method with its simplifying assumptions provided a fast and a simple means of predicting pressure performance for oil or gas flow in a reservoir. **[5]** simulated 21 wells using Vogel's data and developed 13344 IPR curves. To improve the prediction capacity of Vogel's equation, the author introduced a new component "d" to Vogel's expression. **[12]** developed a generalized empirical three-phase IPR similar to Vogel's. He used 4 sets of relative permeability and fluid property data as the basic input for computer model to develop equations to predict inflow performance. The generated relationships are limited by the assumption that the reservoir initially existed at its bubble point pressure. **[10]** developed an IPR equation based on simulation results that attempts to account for the flow efficiency variation caused by rate dependent skin as the flowing bottom-hole pressure changes. **[1]** carried out a mathematical analysis of reservoir productivity by employing a combination of Laplace and Differential transforms. **[13]** studied three-phase inflow performance of oil wells producing oil, water, and gas in a homogenous, bounded reservoir. They developed an IPR correlation based on the basic principle of mass balance using the pseudo-steady state solution. **[2]** carried out an analytical modelling of flow performance of reservoirs under unsteady-state and turbulent conditions. Another theoretical attempt to relate the IPR behaviour with the fundamental flow theories was made by **[4]**. In this model, a second degree polynomial IPR is obtained with a variable coefficient. **[3]** developed an IPR model that outperformed existing models by employing an analytical technique. **[8]** proposed a novel approach to forecasting production rate of dry gas wells using wellhead pressure. Finally, by solving the linear-flow diffusivity equation using Laplace transform, **[9]** introduced a model suitable for predicting the deliverability of aquifers of linear geometry.

2.0 Methodology

The flow chart below describes the sequence of actions taken during the research.

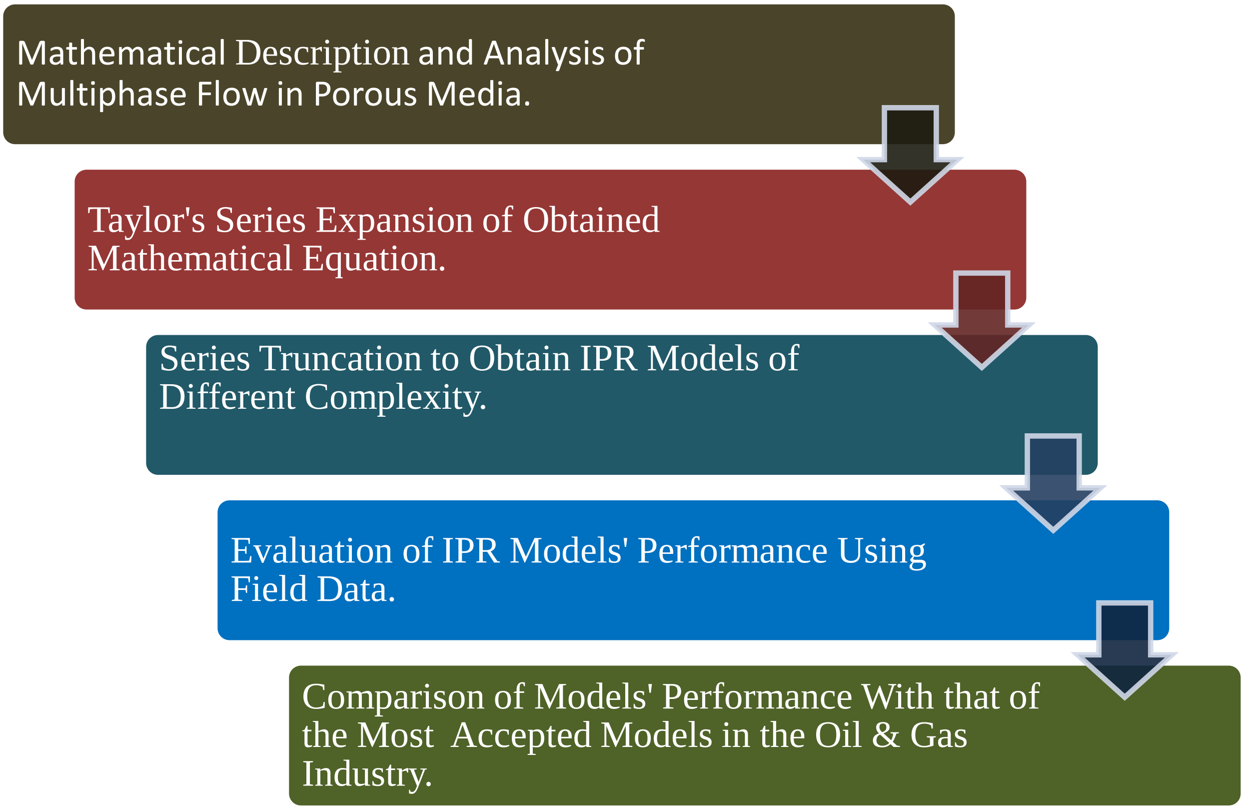

Fig (a): Research Methodology

2.1 Mathematical Description and Analysis of Multiphase Flow in Porous Media

Mathematical model that describes the flow of multiphase fluid in a porous media can be obtained by combining physical principles concerning conservation of mass, Darcy's law for the flow of fluids and an appropriate equation of state.

The general form of these equations for oil and gas flow are:

$$\nabla.\left\{\frac{kK_{ro}}{\mu_o\beta_O}\,\Delta p\right\}=\frac{\partial}{\partial t}\left\{\phi\frac{S_O}{\beta_O}\right\} \quad (1)$$

and

$$\nabla.\left\{\frac{kK_{rg}}{\mu_g\beta_g}+\frac{kK_{ro}R_s}{\mu_o\beta_o}\right\}\Delta p=\frac{\partial}{\partial t}\left\{\phi\frac{S_g}{\beta_g}+\phi\frac{S_oR_s}{\beta_o}\right\} \quad (2)$$

Ignoring capillary effect, gravity and solubility or gas in water.

The above equation is the partial differential equation {P.D.E} for Isotropic, homogeneous, bounded reservoirs producing under boundary-dominated flow conditions.

The second integral of the oil P.D.E for radial flow can be written in terms of the average reservoir pressure as;

$$q_o(t)=\left(\frac{2\pi kh}{\ln\left(\frac{r_e}{r_w}\right)-\frac{3}{4}+s}\right)\int_{P_{wf}}^{p_r}\frac{K_{ro}}{\mu_o\beta_O}\,dP \quad (3)$$

Equation (3) above can be rewritten in general form as

$$q_o(t)=C\int_{P_{wf}}^{p_r}\frac{K_{ro}}{\mu_o\beta_O}\,dP \quad (4)$$

Where C is constant and depend on the geometry of the producing area and flow regime.

If one lets $\Delta P=P_r-P$ (5)

Then, $dP=-d(\Delta P)$ (6)

With this, equation (4) becomes

$$q_o(t)=C\int_0^{\Delta P}\frac{K_{ro}}{\mu_o\beta_O}\,d(\Delta P) \quad (7)$$

Normalizing equation (3.7) by dividing by P_r gives

$$q_o(t)=CP_r\int_0^{\frac{\Delta P}{P_r}}\frac{K_{ro}}{\mu_o\beta_O}\,d\left(\frac{\Delta P}{P_r}\right) \quad (8)$$

At any instant of time during boundary-dominated flow, the flow rate can be written as a function of pressure drop only.

Equation (8) can be expressed about zero in a Taylor series as

$$q_o(\pi)=q_o(0)+\sum_{n=1}^{\infty}\frac{q_o{}^{(n)}(0)}{n!}(\pi)^n \quad (9)$$

Where

$$\pi=\frac{P_r-P}{P_r}=\frac{\Delta P}{P_r}=\frac{P_{wf}}{P_r} \quad (10)$$

By Taylor series, equation (10) can be expanded about zero as;

$$y_{(x)}=y_o\,(x_o)\frac{x-x_o}{0!}+y'(x_o)\frac{(x-x_o)^1}{1!}+y''(x_o)\frac{(x-x_o)^2}{2!}+y'''(x_o)\frac{(x-x_o)^3}{3!}+\ldots\ldots \quad (11)$$

i.e. for y = f(x).

but in this case, q = f (π)

$$q_{(\pi)} = q(\pi_o)\frac{(\pi-\pi_o)^0}{0!} + q'(\pi_o)\frac{(\pi-\pi_o)^1}{1!} + q''(\pi_o)\frac{(\pi-\pi_o)^2}{2!} + q''(\pi_o)\frac{(\pi-\pi_o)^3}{3!} + q'^{v}(\pi_o)\frac{(\pi-\pi_o)^4}{4!} + q^{v}(\pi_o)\frac{(\pi-\pi_o)^5}{5!} + q^{v\prime}(\pi_o)\frac{(\pi-\pi_o)^6}{6!} + \sum \qquad (12)$$

Where ∑ is the error term resulting from truncating the series after 7th term.
In general term to nth number of the term is

$$q_o{}^{(n)}(0) = \frac{CP_r}{n!}\left[\frac{K_{ro}}{\mu_o\beta_O}\right]_{\pi=0}^{(n-1)} \forall\, n \geq 2 \qquad (13)$$

$$and\ \pi = \frac{P_r - P_{wf}}{P_r} = 0 \quad (at\ initial\ state)$$

$$q_o(\pi) = CP_r\left\{\pi\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{\pi^2}{2!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}' + \frac{\pi^3}{3!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}'' + \frac{\pi^4}{4!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}''' + \frac{\pi^5}{5!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}'^{v}\right\} + \varepsilon \qquad (14)$$

Where ε *is the error term of truncating after the* $5th$ *terms.*
Equation (14) allows one to estimate the flow rate for any given flowing pressure at the time the average reservoir pressure = P_r.
To obtain the maximum flow rate, let the wellbore flowing pressure (P_{wf}) = 0, then π *becomes*

$$q_{0max} = CP_r\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right) + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}' + \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}'' + \frac{1}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}''' + \frac{1}{125}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}'^{v} + \frac{1}{125}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}'^{v}\right\} \qquad (15)$$

Vogel suggested that at a given time, the ratio of the oil rate to its maximum rate could be determined from the pressure ratio.

$$\frac{Q_O}{Q_{omax}} = 1 - 0.2\left(\frac{p_{wf}}{p_r}\right) - 0.8\left(\frac{p_{wf}}{p_r}\right)^2 \qquad (16)$$

Using this suggestion, the ratio of equation (14) to equation (15)

$$\frac{q_o}{q_{0max}} = 1 + \frac{C_1}{D}\frac{p_{wf}}{p_r} + \frac{C_2}{D}\left(\frac{p_{wf}}{p_r}\right)^2 + \frac{C_3}{D}\left(\frac{p_{wf}}{p_r}\right)^3 + \frac{C_4}{D}\left(\frac{p_{wf}}{p_r}\right)^4 + \frac{C_5}{D}\left(\frac{p_{wf}}{p_r}\right)^5 + \frac{C_6}{D}\left(\frac{p_{wf}}{p_r}\right)^6 + \frac{C_7}{D}\left(\frac{p_{wf}}{p_r}\right)^7 \qquad (17)$$

2.2 IPR Models Development

By varying the number of terms in equation (17) above, five different IPR models are obtained as follows.

2.2.1 IPR model of degree three (M^3)

$$q_o(\pi) = CP_r\left\{\pi\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{\pi^2}{2!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}' + \frac{\pi^3}{3!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}''\right\}$$

$$q_{0max} = CP_r\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}' + \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}''\right\}$$

Let D = q_{0max}
Using Vogel suggestion.

$$\frac{q_o}{q_{0max}} = 1 + \frac{C_1}{D}\frac{p_{wf}}{p_r} + \frac{C_2}{D}\left(\frac{p_{wf}}{p_r}\right)^2 + \frac{C_3}{D}\left(\frac{p_{wf}}{p_r}\right)^3$$

Where;

$\pi = 1 - \frac{p_{wf}}{p_r}$

$\pi^2 = 1 - 2\frac{p_{wf}}{p_r} + \frac{{p_{wf}}^2}{{p_r}^2}$

$\pi^3 = 1 - 3\frac{p_{wf}}{p_r} + 3\frac{{p_{wf}}^2}{{p_r}^2} - \frac{{p_{wf}}^3}{{p_r}^3}$

Comparing coefficients

$$C_1 = all\ coefficient\ of\ \frac{p_{wf}}{p_r}$$

$$\frac{p_{wf}}{p_r}C_1 = -\frac{p_{wf}}{p_r}\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0}\right\}$$

$$C_1 = -\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0}\right\}$$

$$C_2 = all\ coefficient\ of\ \frac{{p_{wf}}^2}{{p_r}^2}$$

$$\frac{{p_{wf}}^2}{{p_r}^2}C_2 = \left\{\frac{1}{2}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{3}{6}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0}\right\}$$

$$C_2 = \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0}$$

$$C_3 = all\ coefficient\ of\ \frac{{p_{wf}}^3}{{p_r}^3}$$

$$C_3 = -\frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0}$$

Using data from over 54 reservoirs, we have the result below.

Let y = $\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}$

$$\mathrm{Y}(\pi) = 0.326\pi^2 - 0.5173\pi + 0.2568$$

@ $\pi = 0$, $y(\pi) = 0.2568$

$$y'_{(\pi)} = 0.652\pi - 0.5173$$

@ $\pi = 0$, ${y^1}_{\pi} = -0.5173$

$$y''_{(\pi)} = 0.652$$

$$D = y_{(\pi)} + \frac{1}{2}y'_{(\pi)} + \frac{1}{6}y''_{(\pi)}$$

$$D = 0.2568 + \frac{1}{2}(-0.5173) + \frac{1}{6}(0.652)$$

D = 0.1068

$$C_1 = -\left\{y_{(\pi)} + y'_{(\pi)} + \frac{1}{2}y''_{(\pi)}\right\}$$

$$C_1 = -\left\{0.2568 - 0.5173 + \frac{1}{2}(0.652)\right\}$$

C_1 = -0.0655

$$C_2 = -\left\{\frac{1}{2}y'_{(\pi)} + \frac{1}{2}y''_{(\pi)}\right\}$$

$$C_2 = \left\{\frac{1}{2}(-0.5173) + \frac{1}{2}(0.652)\right\}$$

C_2 = -0.0673

$$C_3 = -\left\{\frac{1}{6}y''_{(\pi)}\right\}$$

$$C_3 = -\left\{\frac{1}{6}(0.652)\right\}$$

C_3 = -0.1087

$$\frac{C_1}{D} = \frac{-0.0655}{0.1068} = -0.6133$$

$$\frac{C_2}{D} = \frac{0.0673}{0.1068} = 0.6301$$

$$\frac{C_3}{D} = \frac{-0.1087}{0.1068} = -1.0178$$

Bringing all terms together, we have the result below.

$$\frac{q_o}{q_{omax}} = 1 - 0.6133\frac{P_{wf}}{P_r} + 0.6301\frac{{P_{wf}}^2}{{P_r}^2} - 1.0178\frac{{P_{wf}}^3}{{P_r}^3} \quad (18)$$

2.2.2 IPR model of degree four (M^4)

$$q_o(\pi) = CP_r\left\{\pi\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{\pi^2}{2!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{\pi^3}{3!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0} + \frac{\pi^4}{4!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'''_{\pi=0}\right\}$$

$$q_{0max} = CP_r\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0} + \frac{1}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'''_{\pi=0}\right\}$$

Let D = q_{0max}

Using Vogel suggestion.

$$\frac{q_o}{q_{0max}} = 1 + \frac{C_1}{D}\frac{p_{wf}}{p_r} + \frac{C_2}{D}\left(\frac{p_{wf}}{p_r}\right)^2 + \frac{C_3}{D}\left(\frac{p_{wf}}{p_r}\right)^3 + \frac{C_4}{D}\left(\frac{p_{wf}}{p_r}\right)^4$$

Where;

$\pi = 1 - \frac{p_{wf}}{p_r}$

$\pi^2 = 1 - 2\frac{p_{wf}}{p_r} + \frac{{p_{wf}}^2}{{p_r}^2}$

$\pi^3 = 1 - 3\frac{p_{wf}}{p_r} + 3\frac{{p_{wf}}^2}{{p_r}^2} - \frac{{p_{wf}}^3}{{p_r}^3}$

$\pi^4 = 1 - 4\frac{p_{wf}}{p_r} + 6\frac{{p_{wf}}^2}{{p_r}^2} - 4\frac{{p_{wf}}^3}{{p_r}^3} + \frac{{p_{wf}}^4}{{p_r}^4}$

Comparing coefficients

$$C_1 = all\ coefficient\ of\ \frac{p_{wf}}{p_r}$$

$$\frac{p_{wf}}{p_r}C_1 = -\frac{p_{wf}}{p_r}\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{2}{2!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{3}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0} + \frac{4}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'''_{\pi=0}\right\}$$

$$C_1 = -\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime}\right\}$$

$$C_2 = all\ coefficient\ of\ \frac{{p_{wf}}^2}{{p_r}^2}$$

$$\frac{{p_{wf}}^2}{{p_r}^2}C_2 = \left\{\frac{1}{2}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{3}{6}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{6}{24}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime}\right\}$$

$$C_2 = \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{1}{4}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime}$$

$$C_3 = all\ coefficient\ of\ \frac{{p_{wf}}^3}{{p_r}^3}$$

$$\frac{{p_{wf}}^3}{{p_r}^3}C_3 = -\frac{1}{6}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} - \frac{4}{24}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime}$$

$$C_3 = -\frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} - \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime}$$

$$C_4 = all\ coefficient\ of\ \frac{{p_{wf}}^4}{{p_r}^4}$$

$$C_4 = \frac{1}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime}$$

Using data from over 54 reservoirs, we have the result below.

Let y = $\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}$

$$\mathrm{Y}(\pi) = -0.0915\pi^3 + 0.43\pi^2 - 0.5469\pi + 0.2585$$

$@\ \pi = 0, \qquad y(\pi) = 0.2585$

$$y'_{(\pi)} = -0.2745\pi^2 + 0.86\pi - 0.5469$$

$@\ \pi = 0, \qquad {y^1}_{\pi} = -0.5469$

$$y''_{(\pi)} = -0.549\pi + 0.86$$

$@\ \pi = 0, \qquad y''_{\pi} = 0.86$

$$y'''_{(\pi)} = -0.549$$

$$D = y_{(\pi)} + \frac{1}{2}y'_{(\pi)} + \frac{1}{6}y''_{(\pi)} + \frac{1}{24}y'''_{(\pi)}$$

$$D = 0.2585 + \frac{1}{2}(-0.5469) + \frac{1}{6}(0.86) + \frac{1}{24}(-0.549)$$

D = 0.1054

$$C_1 = -\left\{y_{(\pi)} + y'_{(\pi)} + \frac{1}{2}y''_{(\pi)} + \frac{1}{6}y'''_{(\pi)}\right\}$$

$$C_1 = -\left\{0.2585 - (0.5469) + \frac{1}{2}(0.86) - \frac{1}{6}(0.549)\right\}$$

C_1 = -0.0501

$$C_2 = \left\{\frac{1}{2}y'_{(\pi)} + \frac{1}{2}y''_{(\pi)} + \frac{1}{4}y'''_{(\pi)}\right\}$$

$$C_2 = \left\{\frac{1}{2}(-0.5469) + \frac{1}{2}(0.86) - \frac{1}{4}(-0.549)\right\}$$

C_2 = -0.0192

$$C_3 = -\left\{\frac{1}{6}y''_{(\pi)} + \frac{1}{6}y'''_{(\pi)}\right\}$$

$$C_3 = -\left\{\frac{1}{6}(0.86) + \frac{1}{6}(-0.549)\right\}$$

C_3 = -0.0518

$$C_4 = \left\{\frac{1}{24}y'''_{(\pi)}\right\}$$

$$C_4 = \left\{\frac{1}{24}(-0.549)\right\}$$

C_4 = -0.0229

$$\frac{C_1}{D} = \frac{-0.0501}{0.1054} = -0.4753$$

$$\frac{C_2}{D} = \frac{0.0192}{0.1054} = 0.1822$$

$$\frac{C_3}{D} = \frac{-0.0518}{0.1054} = -0.4915$$

$$\frac{C_4}{D} = \frac{-0.0229}{0.1054} = -0.2173$$

Bringing all terms together, we have the result below.

$$\frac{q_o}{q_{omax}} = 1 - 0.4753\frac{P_{wf}}{P_r} + 0.1822\frac{{P_{wf}}^2}{{P_r}^2} - 0.4915\frac{{P_{wf}}^3}{{P_r}^3} - 0.2173\frac{{P_{wf}}^4}{{P_r}^4} \quad (19)$$

2.2.3 IPR model of degree five (M^5)

$$q_o(\pi) = CP_r\left\{\pi\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{\pi^2}{2!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{\pi^3}{3!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0} + \frac{\pi^4}{4!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'''_{\pi=0} + \frac{\pi^5}{5!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)^{'v}_{\pi=0}\right\}$$

$$q_{0max} = CP_r\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0} + \frac{1}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'''_{\pi=0} + \frac{1}{125}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)^{'v}_{\pi=0}\right\}$$

Let D = q_{0max}

Using Vogel suggestion.

$$\frac{q_o}{q_{0max}} = 1 + \frac{C_1}{D}\frac{p_{wf}}{p_r} + \frac{C_2}{D}\left(\frac{p_{wf}}{p_r}\right)^2 + \frac{C_3}{D}\left(\frac{p_{wf}}{p_r}\right)^3 + \frac{C_4}{D}\left(\frac{p_{wf}}{p_r}\right)^4 + \frac{C_5}{D}\left(\frac{p_{wf}}{p_r}\right)^5$$

Where;

$\pi = 1 - \frac{p_{wf}}{p_r}$

$\pi^2 = 1 - 2\frac{p_{wf}}{p_r} + \frac{{p_{wf}}^2}{{p_r}^2}$

$\pi^3 = 1 - 3\frac{p_{wf}}{p_r} + 3\frac{{p_{wf}}^2}{{p_r}^2} - \frac{{p_{wf}}^3}{{p_r}^3}$

$\pi^4 = 1 - 4\frac{p_{wf}}{p_r} + 6\frac{{p_{wf}}^2}{{p_r}^2} - 4\frac{{p_{wf}}^3}{{p_r}^3} + \frac{{p_{wf}}^4}{{p_r}^4}$

$\pi^5 = 1 - 5\frac{p_{wf}}{p_r} + 10\frac{{p_{wf}}^2}{{p_r}^2} - 10\frac{{p_{wf}}^3}{{p_r}^3} + 5\frac{{p_{wf}}^4}{{p_r}^4} - \frac{{p_{wf}}^4}{{p_r}^4}$

Comparing coefficients

$$C_1 = all\ coefficient\ of\ \frac{p_{wf}}{p_r}$$

$$\frac{p_{wf}}{p_r}C_1 = -\frac{p_{wf}}{p_r}\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{2}{2!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{3}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0} + \frac{4}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'''_{\pi=0} + \frac{5}{120}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)^{'v}_{\pi=0}\right\}$$

$$C_1 = -\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0} + \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'''_{\pi=0} + \frac{1}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)^{'v}_{\pi=0}\right\}$$

$$C_2 = all\ coefficient\ of\ \frac{{p_{wf}}^2}{{p_r}^2}$$

$$\frac{{p_{wf}}^2}{{p_r}^2}C_2 = \left\{\frac{1}{2}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{3}{6}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0} + \frac{6}{24}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0} + \frac{10}{120}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)^{'v}_{\pi=0}\right\}$$

$$C_2 = \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'_{\pi=0} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0} + \frac{1}{4}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'''_{\pi=0} + \frac{1}{12}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)^{'v}_{\pi=0}$$

$$C_3 = all\ coefficient\ of\ \frac{{p_{wf}}^3}{{p_r}^3}$$

$$\frac{{p_{wf}}^3}{{p_r}^3}C_3 = -\frac{1}{6}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0} - \frac{4}{24}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'''_{\pi=0} - \frac{10}{125}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)^{'v}_{\pi=0}$$

$$C_3 = -\frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)''_{\pi=0} - \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'''_{\pi=0} - \frac{1}{12}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)^{'v}_{\pi=0}$$

$$C_4 = all\ coefficient\ of\ \frac{{p_{wf}}^4}{{p_r}^4}$$

$$\frac{{p_{wf}}^4}{{p_r}^4}C_4 = \frac{1}{24}\frac{{p_{wf}}^4}{{p_r}^4}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)'''_{\pi=0} + \frac{5}{120}\frac{{p_{wf}}^4}{{p_r}^4}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)^{'v}_{\pi=0}$$

$$C_5 = all\ coefficient\ of\ \frac{{p_{wf}}^5}{{p_r}^5}$$

$$\frac{{p_{wf}}^5}{{p_r}^5}C_5 = -\frac{1}{120}\frac{{p_{wf}}^5}{{p_r}^5}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)^{'v}_{\pi=0}$$

$$C_5 = -\frac{1}{120}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)^{'v}_{\pi=0}$$

Using data from over 54 reservoirs, we have the result below.

Let y = $\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}$

$$\mathrm{Y}(\pi) = 0.098\pi^4 - 0.2493\pi^3 + 0.5088\pi^2 - 0.5599\pi + 0.259$$

@ $\pi = 0$, $\quad y(\pi) = 0.259$

$$y'_{(\pi)} = 0.3936\pi^3 - 0.7479\pi^2 + 1.0176\pi - 0.5599$$

@ $\pi = 0$, $\quad {y^1}_{\pi} = -0.5599$

$$y''_{(\pi)} = 1.1808\pi^2 - 1.4958\pi + 1.0176$$

$@\ \pi = 0, \qquad y''_{\pi} = 1.0176$

$$y'''_{(\pi)} = 2.3616\pi + 1.4958$$

$@\ \pi = 0, \qquad y'''_{\pi} = 1.4958$

$$y'^{v}_{(\pi)} = 2.3616$$

$$D = y_{(\pi)} + \tfrac{1}{2}y'_{(\pi)} + \tfrac{1}{6}y''_{(\pi)} + \tfrac{1}{24}y'''_{(\pi)} + \tfrac{1}{120}y'^{v}_{(\pi)}$$

$$D = 0.259 + \frac{1}{2}(-0.5599) + \frac{1}{6}(1.0176) + \frac{1}{24}(-1.4958) + \frac{1}{120}(2.3616)$$

D = 0.1061

$$C_1 = -\left\{y_{(\pi)} + y'_{(\pi)} + \frac{1}{2}y''_{(\pi)} + \frac{1}{6}y'''_{(\pi)}\frac{1}{6}y'''_{(\pi)} + \frac{1}{24}y'^{v}_{(\pi)}\right\}$$

$$C_1 = -\left\{0.259 - (0.5599) + \frac{1}{2}(1.0176) - \frac{1}{6}(1.4958) + \frac{1}{24}(2.3616)\right\}$$

C_1 = -0.0570

$$C_2 = \left\{\frac{1}{2}y'_{(\pi)} + \frac{1}{2}y''_{(\pi)} + \frac{1}{4}y'''_{(\pi)} + \frac{1}{12}y'^{v}_{(\pi)}\right\}$$

$$C_2 = \left\{\frac{1}{2}(-0.5599) + \frac{1}{2}(1.0176) + \frac{1}{4}(-1.4958) + \frac{1}{12}(2.3616)\right\}$$

C_2 = 0.0518

$$C_3 = -\left\{\frac{1}{6}y''_{(\pi)} + \frac{1}{6}y'''_{(\pi)} + \frac{1}{12}y'^{v}_{(\pi)}\right\}$$

$$C_3 = -\left\{\frac{1}{6}(1.0176) + \frac{1}{6}(-1.4958) + \frac{1}{12}(2.3616)\right\}$$

C_3 = -0.1171

$$C_4 = \left\{\frac{1}{24}y'''_{(\pi)} + \frac{1}{24}y'^{v}_{(\pi)}\right\}$$

$$C_4 = \left\{\frac{1}{24}(-1.4958) + \frac{1}{24}(2.3616)\right\}$$

C_4 = 0.0361

$$C_5 = \left\{-\frac{1}{120}y'^{v}_{(\pi)}\right\}$$

$$C_5 = \left\{-\frac{1}{120}(2.3616)\right\}$$

C_5 = -0.0197

$$\frac{C_1}{D} = \frac{-0.0570}{0.1061} = -0.5372$$

$$\frac{C_2}{D} = \frac{0.0518}{0.1061} = 0.4822$$

$$\frac{C_3}{D} = \frac{-0.1171}{0.1061} = -1.1037$$

$$\frac{C_4}{D} = \frac{0.0361}{0.1061} = 0.3402$$

$$\frac{C_5}{D} = \frac{-0.0197}{0.1061} = -0.1857$$

Bringing all terms together, we have the result below.

$$\frac{q_o}{q_{omax}} = 1 - 0.5372\frac{P_{wf}}{P_r} + 0.4882\frac{{P_{wf}}^2}{{P_r}^2} - 1.1037\frac{{P_{wf}}^3}{{P_r}^3} + 0.3402\frac{{P_{wf}}^4}{{P_r}^4} - 0.1857\frac{{P_{wf}}^5}{{P_r}^5} \qquad (20)$$

2.2.4 IPR model of degree six (M^6)

$$q_o(\pi) = CP_r\left\{\pi\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{\pi^2}{2!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{\pi^3}{3!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{\pi^4}{4!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{\pi^5}{5!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{\pi^6}{6!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}\right\}$$

$$q_{0max} = CP_r\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{1}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{1}{125}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{1}{720}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}\right\}$$

Let D = q_{0max}
Using Vogel suggestion.

$$\frac{q_o}{q_{0max}} = 1 + \frac{C_1}{D}\frac{p_{wf}}{p_r} + \frac{C_2}{D}\left(\frac{p_{wf}}{p_r}\right)^2 + \frac{C_3}{D}\left(\frac{p_{wf}}{p_r}\right)^3 + \frac{C_4}{D}\left(\frac{p_{wf}}{p_r}\right)^4 + \frac{C_5}{D}\left(\frac{p_{wf}}{p_r}\right)^5 + \frac{C_6}{D}\left(\frac{p_{wf}}{p_r}\right)^6$$

Where;

$\pi = 1 - \frac{p_{wf}}{p_r}$

$\pi^2 = 1 - 2\frac{p_{wf}}{p_r} + \frac{{p_{wf}}^2}{{p_r}^2}$

$\pi^3 = 1 - 3\frac{p_{wf}}{p_r} + 3\frac{{p_{wf}}^2}{{p_r}^2} - \frac{{p_{wf}}^3}{{p_r}^3}$

$\pi^4 = 1 - 4\frac{p_{wf}}{p_r} + 6\frac{{p_{wf}}^2}{{p_r}^2} - 4\frac{{p_{wf}}^3}{{p_r}^3} + \frac{{p_{wf}}^4}{{p_r}^4}$

$\pi^5 = 1 - 5\frac{p_{wf}}{p_r} + 10\frac{{p_{wf}}^2}{{p_r}^2} - 10\frac{{p_{wf}}^3}{{p_r}^3} + 5\frac{{p_{wf}}^4}{{p_r}^4} - \frac{{p_{wf}}^4}{{p_r}^4}$

$\pi^6 = 1 - 6\frac{p_{wf}}{p_r} + 15\frac{{p_{wf}}^2}{{p_r}^2} - 20\frac{{p_{wf}}^3}{{p_r}^3} + 15\frac{{p_{wf}}^4}{{p_r}^4} - 6\frac{{p_{wf}}^5}{{p_r}^5} + \frac{{p_{wf}}^6}{{p_r}^6}$

Comparing coefficients

$$C_1 = all\ coefficient\ of\ \frac{p_{wf}}{p_r}$$

$$\frac{p_{wf}}{p_r}C_1 = -\frac{p_{wf}}{p_r}\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{2}{2!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{3}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{4}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{5}{120}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{6}{720}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}\right\}$$

$$C_1 = -\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{1}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{1}{120}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}\right\}$$

$$C_2 = all\ coefficient\ of\ \frac{{p_{wf}}^2}{{p_r}^2}$$

$$\frac{{p_{wf}}^2}{{p_r}^2}C_2 = \left\{\frac{1}{2}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{3}{6}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{6}{24}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{10}{120}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{15}{720}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}\right\}$$

$$C_2 = \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{1}{4}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{1}{12}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{1}{48}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}$$

$$C_3 = all\ coefficient\ of\ \frac{{p_{wf}}^3}{{p_r}^3}$$

$$\frac{{p_{wf}}^3}{{p_r}^3}C_3 = -\frac{1}{6}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} - \frac{4}{24}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} - \frac{10}{120}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{20}{720}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}$$

$$C_3 = -\frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} - \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} - \frac{1}{12}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} - \frac{1}{36}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}$$

$$C_4 = all\ coefficient\ of\ \frac{{p_{wf}}^4}{{p_r}^4}$$

$$\frac{{p_{wf}}^4}{{p_r}^4}C_4 = \frac{1}{24}\frac{{p_{wf}}^4}{{p_r}^4}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{5}{120}\frac{{p_{wf}}^4}{{p_r}^4}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{15}{720}\frac{{p_{wf}}^4}{{p_r}^4}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}$$

$$C_4 = \frac{1}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{5}{120}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{1}{48}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}$$

$$C_5 = all\ coefficient\ of\ \frac{{p_{wf}}^5}{{p_r}^5}$$

$$\frac{{p_{wf}}^5}{{p_r}^5}C_5 = -\frac{1}{120}\frac{{p_{wf}}^5}{{p_r}^5}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} - \frac{6}{720}\frac{{p_{wf}}^5}{{p_r}^5}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}$$

$$C_5 = -\frac{1}{120}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} - \frac{1}{120}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}$$

$$C_6 = all\ coefficient\ of\ \frac{{p_{wf}}^6}{{p_r}^6}$$

$$\frac{{p_{wf}}^6}{{p_r}^6}C_6 = -\frac{1}{720}\frac{{p_{wf}}^6}{{p_r}^6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}$$

$$C_6 = -\frac{1}{720}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}$$

Using data from over 54 reservoirs, we have the result below.

Let y = $\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}$

$$Y(\pi) = -1.2435\pi^5 + 2.6059\pi^4 - 2.017\pi^3 + 1.0251\pi^2 - 0.6164\pi + 0.2603$$

@ $\pi = 0$, $\quad y(\pi) = 0.2603$

$$y'_{(\pi)} = -6.2175\pi^4 + 10.4236\pi^3 - 6.051\pi^2 + 2.0502\pi - 0.6164$$

$@\,\pi = 0, \qquad y^1{}_{\pi} = -0.6164$

$$y''_{(\pi)} = -24.87\pi^3 + 31.2708\pi^2 - 12.102\pi + 2.0502$$

$@\,\pi = 0, \qquad y''_{\pi} = 2.0502$

$$y'''_{(\pi)} = -74.61\pi^2 + 62.5416\pi - 12.102$$

$@\,\pi = 0, \qquad y'''_{\pi} = -12.102$

$$y'^{v}_{(\pi)} = -149.22\pi + 62.5416$$

$@\,\pi = 0, \qquad y'^{v}_{\pi} = 62.5416$

$$y^{v}_{(\pi)} = -149.22$$

$$D = y_{(\pi)} + \tfrac{1}{2}y'_{(\pi)} + \tfrac{1}{6}y''_{(\pi)} + \tfrac{1}{24}y'''_{(\pi)} + \tfrac{1}{120}y'^{v}_{(\pi)} + \tfrac{1}{720}y^{v}_{(\pi)}$$

$$D = 0.2603 + \frac{1}{2}(-0.6164) + \frac{1}{6}(2.0502\,) + \frac{1}{24}(-12.102) + \frac{1}{120}(62.5416\,) + \frac{1}{720}(-149.22)$$

D = 0.1035

$$C_1 = -\left\{y_{(\pi)} + y'_{(\pi)} + \frac{1}{2}y''_{(\pi)} + \frac{1}{6}y'''_{(\pi)} + \frac{1}{24}y'^{v}_{(\pi)} + \frac{1}{120}y^{v}_{(\pi)}\right\}$$

$$C_1 = -\left\{0.2603 - (0.6164) + \frac{1}{2}(2.0502) - \frac{1}{6}(12.102) + \frac{1}{24}(62.5416) + \frac{1}{120}(-149.22)\right\}$$

C_1 = -0.0144

$$C_2 = \left\{\frac{1}{2}y'_{(\pi)} + \frac{1}{2}y''_{(\pi)} + \frac{1}{4}y'''_{(\pi)} + \frac{1}{12}y'^{v}_{(\pi)} + \frac{1}{48}y'^{v}_{(\pi)}\right\}$$

$$C_2 = \left\{\frac{1}{2}(-0.6164) + \frac{1}{2}(2.0502) + \frac{1}{4}(-12.102) + \frac{1}{12}(62.5416) + \frac{1}{48}(-149.22)\right\}$$

C_2 = -0.2056

$$C_3 = -\left\{\frac{1}{6}y''_{(\pi)} + \frac{1}{6}y'''_{(\pi)} + \frac{1}{12}y'^{v}_{(\pi)} + \frac{1}{36}y^{v}_{(\pi)}\right\}$$

$$C_3 = -\left\{\frac{1}{6}(2.0502) + \frac{1}{6}(-12.102) + \frac{1}{12}(62.5416) + \frac{1}{36}(-149.22)\right\}$$

C_3 = 0.6085

$$C_4 = \left\{\frac{1}{24}y'''_{(\pi)} + \frac{1}{24}y'^{v}_{(\pi)} + \frac{1}{48}y^{v}_{(\pi)}\right\}$$

$$C_4 = \left\{\frac{1}{24}(-12.102) + \frac{1}{24}(62.5416) + \frac{1}{48}(-149.22)\right\}$$

C_4 = -1.0071

$$C_5 = \left\{-\frac{1}{120}y'^{v}_{(\pi)} - \frac{1}{120}y^{v}_{(\pi)}\right\}$$

$$C_5 = \left\{-\frac{1}{120}(62.5416) - \frac{1}{120}(-149.22)\right\}$$

C_5 = 0.7223

$$C_6 = \left\{\frac{1}{720}y^{v}_{(\pi)}\right\}$$

$$C_6 = \left\{\frac{1}{720}(-149.22)\right\}$$

C_6 = -0.2073

$$\frac{C_1}{D} = \frac{-0.0144}{0.1035} = -0.1391$$
$$\frac{C_2}{D} = \frac{-0.2056}{0.1035} = -1.9865$$
$$\frac{C_3}{D} = \frac{0.6085}{0.1035} = 5.8792$$
$$\frac{C_4}{D} = \frac{-1.0071}{0.1035} = -9.7304$$
$$\frac{C_5}{D} = \frac{0.7223}{0.1035} = 6.9787$$
$$\frac{C_6}{D} = \frac{-0.2073}{0.1035} = -2.0029$$

Bringing all terms together, we have the result below.

$$\frac{q_o}{q_{omax}} = 1 - 0.1391\frac{P_{wf}}{P_r} - 1.9865\frac{{P_{wf}}^2}{{P_r}^2} + 5.8792\frac{{P_{wf}}^3}{{P_r}^3} - 9.7304\frac{{P_{wf}}^4}{{P_r}^4} + 6.9787\frac{{P_{wf}}^5}{{P_r}^5} - 2.0029\frac{{P_{wf}}^6}{{P_r}^6}$$
(21)

2.2.5 IPR model of degree seven (M^7)

$$q_o(\pi) = CP_r\left\{\pi\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{\pi^2}{2!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}' + \frac{\pi^3}{3!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}'' + \frac{\pi^4}{4!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}''' + \frac{\pi^5}{5!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{\pi^6}{6!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} + \frac{\pi^7}{7!}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}\right\}$$

$$q_{0max} = CP_r\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}' + \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}'' + \frac{1}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}''' + \frac{1}{125}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{1}{720}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} + \frac{1}{5040}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}\right\}$$

Let D = q_{0max}

Using Vogel suggestion.

$$\frac{q_o}{q_{0max}} = 1 + \frac{C_1}{D}\frac{p_{wf}}{p_r} + \frac{C_2}{D}\left(\frac{p_{wf}}{p_r}\right)^2 + \frac{C_3}{D}\left(\frac{p_{wf}}{p_r}\right)^3 + \frac{C_4}{D}\left(\frac{p_{wf}}{p_r}\right)^4 + \frac{C_5}{D}\left(\frac{p_{wf}}{p_r}\right)^5 + \frac{C_6}{D}\left(\frac{p_{wf}}{p_r}\right)^6 + \frac{C_7}{D}\left(\frac{p_{wf}}{p_r}\right)^7$$

Where;

$$\pi = 1 - \frac{p_{wf}}{p_r}$$

$$\pi^2 = 1 - 2\frac{p_{wf}}{p_r} + \frac{{p_{wf}}^2}{{p_r}^2}$$

$$\pi^3 = 1 - 3\frac{p_{wf}}{p_r} + 3\frac{{p_{wf}}^2}{{p_r}^2} - \frac{{p_{wf}}^3}{{p_r}^3}$$

$$\pi^4 = 1 - 4\frac{p_{wf}}{p_r} + 6\frac{{p_{wf}}^2}{{p_r}^2} - 4\frac{{p_{wf}}^3}{{p_r}^3} + \frac{{p_{wf}}^4}{{p_r}^4}$$

$$\pi^5 = 1 - 5\frac{p_{wf}}{p_r} + 10\frac{{p_{wf}}^2}{{p_r}^2} - 10\frac{{p_{wf}}^3}{{p_r}^3} + 5\frac{{p_{wf}}^4}{{p_r}^4} - \frac{{p_{wf}}^4}{{p_r}^4}$$

$$\pi^6 = 1 - 6\frac{p_{wf}}{p_r} + 15\frac{{p_{wf}}^2}{{p_r}^2} - 20\frac{{p_{wf}}^3}{{p_r}^3} + 15\frac{{p_{wf}}^4}{{p_r}^4} - 6\frac{{p_{wf}}^5}{{p_r}^5} + \frac{{p_{wf}}^6}{{p_r}^6}$$

$$\pi^7 = 1 - 7\frac{p_{wf}}{p_r} + 21\frac{{p_{wf}}^2}{{p_r}^2} - 35\frac{{p_{wf}}^3}{{p_r}^3} + 35\frac{{p_{wf}}^4}{{p_r}^4} - 21\frac{{p_{wf}}^5}{{p_r}^5} + 7\frac{{p_{wf}}^6}{{p_r}^6} - \frac{{p_{wf}}^7}{{p_r}^7}$$

Comparing coefficients

$$C_1 = all\ coefficient\ of\ \frac{p_{wf}}{p_r}$$

$$\frac{p_{wf}}{p_r}C_1 = -\frac{p_{wf}}{p_r}\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \frac{2}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{3}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{4}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{5}{120}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{6}{720}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} + \frac{7}{5040}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}\right\}$$

$$C_1 = -\left\{\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0} + \left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{1}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{1}{120}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} + \frac{1}{720}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}\right\}$$

$$C_2 = all\ coefficient\ of\ \frac{{p_{wf}}^2}{{p_r}^2}$$

$$\frac{{p_{wf}}^2}{{p_r}^2}C_2 = \left\{\frac{1}{2}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{3}{6}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{6}{24}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{10}{120}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{15}{720}\frac{{p_{wf}}^2}{{p_r}^2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} + \frac{21}{5040}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}\right\}$$

$$C_2 = \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime} + \frac{1}{2}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} + \frac{1}{4}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{1}{12}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{1}{48}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} + \frac{1}{240}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}$$

$$C_3 = all\ coefficient\ of\ \frac{{p_{wf}}^3}{{p_r}^3}$$

$$\frac{{p_{wf}}^3}{{p_r}^3}C_3 = -\frac{1}{6}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} - \frac{4}{24}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} - \frac{10}{120}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{20}{720}\frac{{p_{wf}}^3}{{p_r}^3}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} + \frac{35}{5040}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}$$

$$C_3 = -\frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime} - \frac{1}{6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} - \frac{1}{12}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} - \frac{1}{36}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} - \frac{1}{144}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}$$

$$C_4 = all\ coefficient\ of\ \frac{{p_{wf}}^4}{{p_r}^4}$$

$$\frac{{p_{wf}}^4}{{p_r}^4}C_4 = \frac{1}{24}\frac{{p_{wf}}^4}{{p_r}^4}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{5}{120}\frac{{p_{wf}}^4}{{p_r}^4}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{15}{720}\frac{{p_{wf}}^4}{{p_r}^4}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} + \frac{35}{5040}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}$$

$$C_4 = \frac{1}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime\prime\prime} + \frac{1}{24}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} + \frac{1}{48}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} + \frac{1}{144}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}$$

$$C_5 = all\ coefficient\ of\ \frac{{p_{wf}}^5}{{p_r}^5}$$

$$\frac{{p_{wf}}^5}{{p_r}^5}C_5 = -\frac{1}{120}\frac{{p_{wf}}^5}{{p_r}^5}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} - \frac{6}{720}\frac{{p_{wf}}^5}{{p_r}^5}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} - \frac{21}{5040}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}$$

$$C_5 = -\frac{1}{120}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{\prime v} - \frac{1}{120}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} - \frac{1}{240}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}$$

$$C_6 = all\ coefficient\ of\ \frac{{p_{wf}}^6}{{p_r}^6}$$

$$\frac{{p_{wf}}^6}{{p_r}^6} C_6 = \frac{1}{720}\frac{{p_{wf}}^6}{{p_r}^6}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} + \frac{7}{5040}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}$$

$$C_6 = \frac{1}{720}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v} + \frac{1}{720}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}$$

$$C_7 = all\ coefficient\ of\ \frac{{p_{wf}}^7}{{p_r}^7}$$

$$\frac{{p_{wf}}^7}{{p_r}^7} C_7 = -\frac{1}{5040}\frac{{p_{wf}}^7}{{p_r}^7}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v\prime}$$

$$C_7 = \frac{1}{5040}\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}^{v}$$

Using data from over 54 reservoirs, we have the result below.

Let y = $\left(\frac{K_{ro}}{\mu_o\beta_O}\right)_{\pi=0}$

$$\mathrm{Y}(\pi) = 7.3849\pi^6 - 19.129\pi^5 + 18.796\pi^4 - 8.786\pi^3 + 2.3271\pi^2 - 0.7145\pi + 0.2619$$

@ $\pi = 0, \qquad y(\pi) = 0.2619$

$$y'_{(\pi)} = 44.3094\pi^5 - 95.645\pi^4 + 75.184\pi^3 - 26.358\pi^2 + 4.6542\pi - 0.7145$$

@ $\pi = 0, \qquad {y^1}_\pi = -0.7145$

$$y''_{(\pi)} = 221.547\pi^4 - 382.58\pi^3 + 225.552\pi^2 - 52.716\pi + 4.6542$$

@ $\pi = 0, \qquad y''_\pi = 4.6542$

$$y'''_{(\pi)} = 886.188\pi^3 - 1147.74\pi^2 + 451.104\pi - 52.716$$

@ $\pi = 0, \qquad y'''_\pi = -52.716$

$$y'^{v}_{(\pi)} = 2658.564\pi^2 - 2295.48\pi + 451.104$$

@ $\pi = 0, \qquad y'^{v}_\pi = 451.104$

$$y^{v}_{(\pi)} = 5317.128\pi - 2295.48$$

@ $\pi = 0, \qquad y^{v}_\pi = -2295.48$

$$y^{v\prime}_{(\pi)} = 5317.128$$

$$D = y_{(\pi)} + \tfrac{1}{2}y'_{(\pi)} + \tfrac{1}{6}y''_{(\pi)} + \tfrac{1}{24}y'''_{(\pi)} + \tfrac{1}{120}y'^{v}_{(\pi)} + \tfrac{1}{720}y^{v}_{(\pi)} + \tfrac{1}{5040}y^{v\prime}_{(\pi)}$$

$$D = 0.2619 + \frac{1}{2}(-0.7145) + \frac{1}{6}(4.6542) + \frac{1}{24}(-52.716) + \frac{1}{120}(451.104) + \frac{1}{720}(-2295.48) + \frac{1}{5040}(5317.128)$$

D = 0.1097

$$C_1 = -\left\{y_{(\pi)} + y'_{(\pi)} + \frac{1}{2}y''_{(\pi)} + \frac{1}{6}y'''_{(\pi)} + \frac{1}{24}y'^{v}_{(\pi)} + \frac{1}{120}y^{v}_{(\pi)} + \frac{1}{720}y^{v'}_{(\pi)}\right\}$$

$$C_1 = -\left\{0.2619 + (-0.7145) + \frac{1}{2}(4.6542) + \frac{1}{6}(-52.716) + \frac{1}{24}(451.104) + \frac{1}{120}(-2295.48) + \frac{1}{720}(5317.128)\right\}$$

C_1 = -0.1404

$$C_2 = \left\{\frac{1}{2}y'_{(\pi)} + \frac{1}{2}y''_{(\pi)} + \frac{1}{4}y'''_{(\pi)} + \frac{1}{12}y'^{v}_{(\pi)} + \frac{1}{48}y^{v}_{(\pi)} + \frac{1}{240}y^{v'}_{(\pi)}\right\}$$

$$C_2 = \left\{\frac{1}{2}(-0.7145) + \frac{1}{2}(4.6542) + \frac{1}{4}(-52.716) + \frac{1}{12}(451.104) + \frac{1}{48}(-2295.48) + \frac{1}{240}(5317.128)\right\}$$

C_2 = 0.715

$$C_3 = -\left\{\frac{1}{6}y''_{(\pi)} + \frac{1}{6}y'''_{(\pi)} + \frac{1}{12}y'^{v}_{(\pi)} + \frac{1}{36}y^{v}_{(\pi)} + \frac{1}{144}y^{v'}_{(\pi)}\right\}$$

$$C_3 = -\left\{\frac{1}{6}(4.6542) + \frac{1}{6}(-52.716) + \frac{1}{12}(451.104) + \frac{1}{36}(-2295.48) + \frac{1}{144}(5317.128)\right\}$$

C_3 = -2.7429

$$C_4 = \left\{\frac{1}{24}y'''_{(\pi)} + \frac{1}{24}y'^{v}_{(\pi)} + \frac{1}{48}y^{v}_{(\pi)} + \frac{1}{144}y^{v'}_{(\pi)}\right\}$$

$$C_4 = \left\{\frac{1}{24}(-52.716) + \frac{1}{24}(451.104) + \frac{1}{48}(-2295.48) + \frac{1}{144}(5317.128)\right\}$$

C_4 = 5.7015

$$C_5 = \left\{-\frac{1}{120}y'^{v}_{(\pi)} - \frac{1}{120}y^{v}_{(\pi)} - \frac{1}{240}y^{v'}_{(\pi)}\right\}$$

$$C_5 = \left\{-\frac{1}{120}(451.104) - \frac{1}{120}(-2295.48) - \frac{1}{144}(5317.128)\right\}$$

C_5 = -6.7849

$$C_6 = \left\{\frac{1}{720}y^{v}_{(\pi)} + \frac{1}{720}y^{v'}_{(\pi)}\right\}$$

$$C_6 = \left\{\frac{1}{720}(-2295.48) + \frac{1}{720}(5317.128)\right\}$$

C_6 = 4.1967

$$C_7 = -\left\{\frac{1}{5040}y^{v'}_{(\pi)}\right\}$$

$$C_7 = -\left\{\frac{1}{5040}(5317.128)\right\}$$

C_7 = -1.0550

$$\frac{C_1}{D} = \frac{-0.1404}{0.1097} = -1.2799$$

$$\frac{C_2}{D} = \frac{0.715}{0.1097} = 6.5178$$

$$\frac{C_3}{D} = \frac{-2.7429}{0.1097} = -25.0036$$

$$\frac{C_4}{D} = \frac{5.7015}{0.1097} = 51.9736$$
$$\frac{C_5}{D} = \frac{-6.7849}{0.1097} = -61.8496$$
$$\frac{C_6}{D} = \frac{4.1967}{0.1097} = 38.2562$$
$$\frac{C_7}{D} = \frac{-1.0550}{0.1097} = -9.6171$$

Bringing all terms together, we have the result below.

$$\frac{q_o}{q_{omax}} = 1 - 1.2799\frac{P_{wf}}{P_r} + 6.5178\frac{{P_{wf}}^2}{{P_r}^2} - 25.0036\frac{{P_{wf}}^3}{{P_r}^3} + 51.9736\frac{{P_{wf}}^4}{{P_r}^4} - 61.8496\frac{{P_{wf}}^5}{{P_r}^5} + 38.2562\frac{{P_{wf}}^6}{{P_r}^6} - 9.6171\frac{{P_{wf}}^7}{{P_r}^7}$$
(22)

3.0 Analysis of Models' Performance

Using data from 54 reservoirs, the performance/validity of each of the IPR models introduced in the previous section is evaluated by comparing the performance of each of the models with that of the two widely accepted models in the oil and gas industry.

3.1 Comparison with Wiggins' Model

In this section, the performance of each of the models is compared with that of Wiggins using graphical illustration and a statistical parameter.

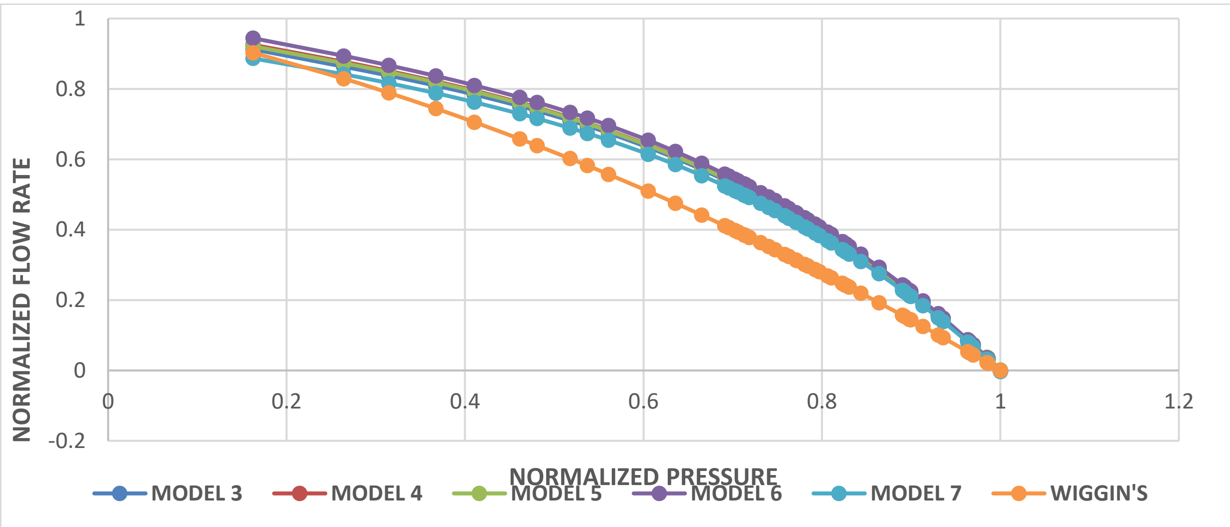

Fig (b): Comparison of models' performance with Wiggins'

Table 1: Comparison of models' performance with Wiggins' using statistical parameter.

Models	Wiggins	M^3	M^4	M^5	M^6	M^7
R^2	0.908	0.978	0.988	0.993	0.995	0.998

3.2 Comparison with Vogel's Model

In this section, the performance of each of the models is compared with that of Vogel using graphical illustration and a statistical parameter.

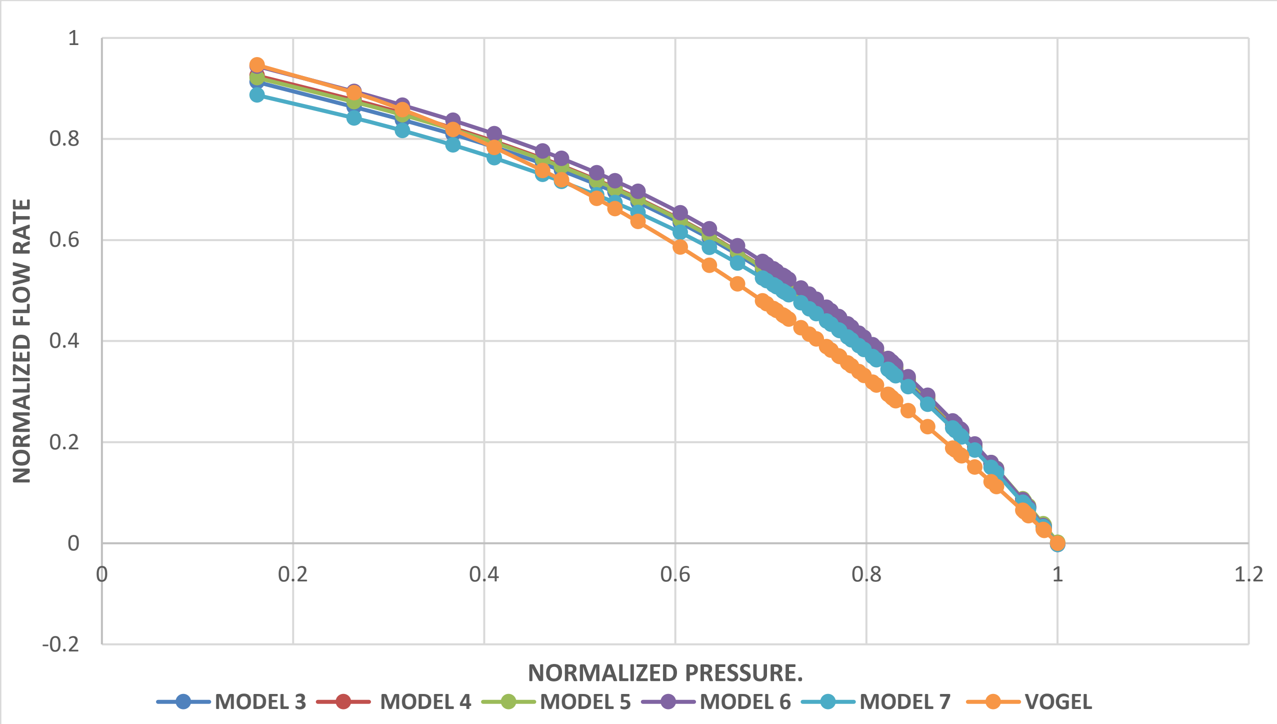

Fig (c): Comparison of models' performance with Vogel's

Table 2: Comparison of models' performance with Vogel's using statistical parameter.

Models	Vogel	M^3	M^4	M^5	M^6	M^7
R^2	0.928	0.978	0.988	0.993	0.995	0.998

4.0 Conclusion and Recommendations

A rigorous mathematical analysis of fluids influx into wells drilled into multiphase flow hydrocarbon reservoirs has been carried out. It proved to be an adventurous and intellectually interesting journey. The pseudo-steady state solution of the Partial Differential Equation (PDE) governing multiphase flow in homogenous and isotropic porous media was obtained via Laplace Transform. Furthermore, the obtained solution was expanded using Taylor's series expansion method in order to obtain a form that is suitable for forecasting production from hydrocarbon reservoirs. By considering different number of terms in the Taylor's series form of the solution, five different Inflow Performance Relationship (IPR) models were obtained. As evident in the results above, the performance of these models increases as the number of terms in the Taylor's series expansion increases. This is an expected result because as the number of terms in the Taylor's series form of the solution increases, the Taylor's series form approximates the exact analytical solution more closely. Although the accuracy of the models increases as the number of terms in the Taylor's series form increases, it can be seen from the performance metric above that beyond the fifth term the incremental improvement in performance becomes negligible. We can therefore conclude that $\boldsymbol{M^4}$, which is a polynomial of degree 4 in $\frac{P_{wf}}{P_r}$ is the optimum model for forecasting production from multiphase flow reservoirs. This is due to the fact that further increase in model's complexity by including more terms in the Taylor's series expansion does not translate to a significant improvement in performance. Furthermore, it is evident from the performance analysis above that the models presented in this paper outperform the two widely accepted models in the oil and gas industry. Therefore, these models (especially $\boldsymbol{M^4}$) are highly recommended for forecasting production from multiphase flow reservoirs as they guarantee a very high degree of accuracy.

1. Declarations

1.1. Author Contributions

Conceptualization, **T.S.A** and **H.O.W**.; methodology, **T.S.A**.; software, **H.O.W**.; validation, **T.S.A** and **H.O.W**; formal analysis, **T.S.A**.; writing—original draft preparation, **T.S.A**.; writing—review and editing, **T.S.A**.; All authors have read and agreed to the published version of the manuscript.

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1.3. Declaration of Competing Interest

The author declares that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors

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