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ABOUT CALCULATING FRACTIONAL DERIVATIVES AND INTEGRAL USING MAPLE

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Abstract
This article provides information about the concepts of fractional derivatives and fractional integrals, as well as some applications.

Keywords: special function, gamma function, fractional derivatives, fractional integrals, fractional order differential equation.

Derivatives and integrals of order K century and their to applications circle field present in the day the most fast developing scientific from fields is one That's why for to students this concepts about knowledge to give sure and natural sciences in teaching current is one of the issues . From this out of this at work Derivatives and integrals of order k asr to the students and their to applications circle theoretical data we give

Fractional derivatives and integrals appear to be circle historical and theoretical data [1-3] from the literature , applications and received scientific results from works [4]. and them given books in the list from work to find can _

If $\varphi(x) \in L_1(a,b)$ if so _

$$\left(I_{a+}^{\alpha}\varphi\right)(x) = \frac{1}{\Gamma(\alpha)} \int_a^x \frac{\varphi(t)}{\left(x-t\right)^{1-\alpha}} dt, \quad x > a, \quad (1)$$

and

$$\left(I_{b-}^{\alpha}\varphi\right)(x) = \frac{1}{\Gamma(\alpha)} \int_x^b \frac{\varphi(t)}{\left(t-x\right)^{1-\alpha}} dt, \quad x < b, \quad (2)$$

in appearance integrals suitable respectively Riemann is the left-hand side of Liouville and right bilaterally α - fraction in order integral it is called on the ground $\alpha > 0$, $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$ - gamma function . Gamma function values for tables created _ them from the Internet or , gamma functions illuminated from the literature to find can _

is known that

$$\frac{1}{\Gamma(\alpha)} \int_a^x \frac{\varphi(t)}{\left(x-t\right)^{1-\alpha}} dt = f(x), \; x > a \quad (3)$$

and

$$\frac{1}{\Gamma(\alpha)} \int_x^b \frac{\varphi(t)}{\left(t-x\right)^{1-\alpha}} dt = f(x), \; x < b \quad (4)$$

in appearance equations are Abel's equations called _ $0 < \alpha < 1$ when suitable respectively this

$$\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_a^x\frac{f(t)}{(x-t)^{\alpha}}dt \quad (5)$$

and

$$\varphi(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_x^b\frac{f(t)}{(t-x)^{\alpha}}dt \quad (6)$$

to solutions have _ Apparently since , (1) and (2) are fractions in order integrals are the left side of Abel's equations in the form (3) and (4). with on top of each other falls _

Fraction in order derivative fraction in order to the integral reverse action as let's find out . Of this for above given Abel's equation from solutions we use

$[a,b]$ in cross section given $f(x)$ function for the following

$$\left(D_{a+}^{\alpha}f\right)(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_a^x\frac{f(t)}{(x-t)^{\alpha}}dt\,, \quad (7)$$

and

$$\left(D_{b-}^{\alpha}f\right)(x)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_x^b\frac{f(t)}{(t-x)^{\alpha}}dt,\quad 0<\alpha<1 \quad (8)$$

formulas suitable respectively Riemann is the left-hand side of Liouville and right bilaterally α - a fraction in order derivative is called

To the above importance if we give it , it's a fraction the concept of order integral optional $\alpha>0$ for was determined . Fraction in order derivative while only $0<\alpha<1$ for was determined . Now fraction in order derivative $\alpha\geq 1$ for how to be determined we will show . Of this for α number $[\alpha]$ - whole part and $\{\alpha\}$- fraction from the part we use Description according to $0<\{\alpha\}<1$ and $\alpha=[\alpha]+\{\alpha\}$.

If α - is an integer , then α - fraction in order derivative as simple differentiation is understood , that is

$$D_{a+}^{\alpha}=\left(\frac{d}{dx}\right)^{\alpha},\ D_{b-}^{\alpha}=\left(-\frac{d}{dx}\right)^{\alpha},\ \alpha=1,2,3,\ldots$$

If α - is not an integer , then $D_{a+}^{\alpha}, D_{b-}^{\alpha}$ s as follows let's find out

$$\left(D_{a+}^{\alpha}f\right)(x)=\left(\frac{d}{dx}\right)^{[\alpha]}\left(D_{a+}^{\{\alpha\}}f\right)(x)=\left(\frac{d}{dx}\right)^{[\alpha]+1}\left(I_{a+}^{1-\{\alpha\}}f\right)(x),$$

$$\left(D_{b-}^{\alpha}f\right)(x)=\left(-\frac{d}{dx}\right)^{[\alpha]}\left(D_{b-}^{\{\alpha\}}f\right)(x)=\left(-\frac{d}{dx}\right)^{[\alpha]+1}\left(I_{b-}^{1-\{\alpha\}}f\right)(x)\,.$$

Above to the said based on , optional α - fraction in order derivative common without as follows let's find out .

$[a,b]$ in cross section given $f(x)$ function for the following

$$\left(D_{a+}^{\alpha} f\right)(x)=\frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^{n}\int_{a}^{x}\frac{f(t)}{(x-t)^{\alpha-n+1}}dt, \quad (9)$$

and

$$\left(D_{b-}^{\alpha} f\right)(x)=\frac{(-1)^{n}}{\Gamma(n-\alpha)}\left(\frac{d}{dx}\right)^{n}\int_{x}^{b}\frac{f(t)}{(t-x)^{\alpha-n+1}}dt, \quad (10)$$

expressions suitable respectively Riemann is the left-hand side of Liouville and right bilaterally α - a fraction in order derivative it is called on the ground $n=[\alpha]+1,\ \alpha>0$.

Some examples we bring 1) $f(x)=x^3$ let it be $f(x)$ optional $[a,b]$ in cross section determined . This function optional $[a,b]$ in cross section $\alpha=2$ whole in order derivative $6x$ to equal to This the result $[0,1]$ between formulas (9) and (10). through we will check .$[\alpha]=2$ that it was for $n=[\alpha]+1=3$ will be Formulas (9) and (10). according to account Maple math works system using we will do it .

$$> \frac{1}{\mathrm{GAMMA}(3-2)}\cdot diff\left(int\left(\frac{t^3}{(x-t)^{2-3+1}}, t=0..x\right), x\$3\right);$$

$$6\,x$$

$$\frac{1}{\mathrm{GAMMA}(3-2)}\cdot\left(-diff\left(int\left(\frac{t^3}{(t-x)^{2-3+1}}, t=x..1\right), x\$3\right)\right);$$

$$6\,x$$

Apparently as it is left- sided and right bilaterally derivatives too $6x$ to equal to Now $\alpha=2{,}5$ let it be let's say In that case $n=[\alpha]+1=3$.$[0,1]$ between formulas (9) and (10). according to account Maple math works system using we do [5].

$$> \frac{1}{\mathrm{GAMMA}(3-2.5)}\cdot diff\left(int\left(\frac{t^3}{(x-t)^{2.5-3+1}}, t=0..x\right), x\$3\right);$$

$$6.770275002\sqrt{x}$$

$$\frac{1}{\mathrm{GAMMA}(3-2.5)}\cdot\left(-diff\left(int\left(\frac{t^3}{(t-x)^{2.5-3+1}}, t=x..1\right), x\$3\right)\right);$$

$$\frac{0.1934364287x^3}{(1.-1.x)^{5/2}}+\frac{1.160618572x^2}{(1.-1.x)^{3/2}}+\frac{4.642474288x}{\sqrt{1.-1.x}}$$
$$-3.094982858\sqrt{1.-1.x}+\frac{0.09671821430x^2}{(1.-1.x)^{5/2}}$$
$$+\frac{0.3868728573x}{(1.-1.x)^{3/2}}+\frac{0.7737457143}{\sqrt{1.-1.x}}+\frac{0.07253866075x}{(1.-1.x)^{5/2}}$$
$$+\frac{0.1450773214}{(1.-1.x)^{3/2}}+\frac{0.06044888392}{(1.-1.x)^{5/2}}$$

Received from the results it seems that α whole otherwise , it is left- sided and right bilaterally derivatives equal to it's not . From this except different intervals for results different will be

References

1. Самко С.Г., Килбас А.А., Маричев О.И. Интегралы и производные дробного порядка и некоторые их приложения. -Минск: Наука и техника, 1987, -687 с.
2. Псху А.В. Уравнения в частных производных дробного порядка. М.: Наука, 2005, - 199 с.
3. Килбас А.А. Теория и приложения дифференциальных уравнений дробного порядка(Курс лекций) //Методологическая школа-конференция "Математическая физика и нанатехнологии" посвященная 40-летию возрождения Самарского государственного университета (Самара, 4-9 октября 2009 года), - 121 с.
4. Турметов Б.Х. О разрешимости одной краевой задачи для неоднородного полигармонического уравнения с граничным оператором дробного порядка.//Уфимский математический журнал. Том 8. №3, - 2016, С. 160-175.
5. Говорухин В., Цибулин В. Компьютер в математическом исследовании. «Питер», -2001, 624 с.: ил.